331. Proposed by T. H. GRONWALL, Chicago, Illinois.

To show that:

(1) $\frac{d^n}{dx^n}\left(\frac{\sin x}{x}\right) = \frac{1}{x^{n+1}}\int_0^x y^n \sin\left(y + \frac{n+1}{2}\pi\right) dy,$

(2) $\frac{d^n}{dx^n}\left(\frac{1-\cos x}{x}\right) = \frac{1}{x^{n+1}}\int_0^x y^n \sin\left(y + \frac{n}{2}\pi\right) dy.$

SOLUTION BY ELIJAH SWIFT, Princeton, N. J.

Assume that the formula is true for $n - 1$.

Then

$$\frac{d^{n-1}}{dx^{n-1}}\left(\frac{\sin x}{x}\right) = \frac{1}{x^n}\int_0^x y^{n-1} \sin\left(y + \frac{n}{2}\pi\right) dy.$$

Differentiating both sides and using the formula we have to prove

$$(A) \quad \frac{1}{x^{n+1}}\int_0^x y^n \sin\left(y + \frac{n+1}{2}\pi\right) dy = -\frac{n}{x^{n+1}}\int_0^x y^{n-1} \sin\left(y + \frac{n}{2}\pi\right) dy + \frac{1}{x^n}\cdot x^{n-1}\sin\left(x + \frac{n}{2}\pi\right).$$

or

$$(B) \quad \int_0^x y^n \sin\left(y + \frac{n+1}{2}\pi\right) dy = -n\int_0^x y^{n-1} \sin\left(y + \frac{n\pi}{2}\right) dy + x^n \sin\left(x + \frac{n}{2}\pi\right).$$

Differentiating B as to x,

$$(C) \quad x^n \sin\left(x + \frac{n+1}{2}\pi\right) = -nx^{n-1}\sin\left(x + \frac{n\pi}{2}\right) + nx^{n-1}\sin\left(x + \frac{n\pi}{2}\right) + x^n \cos\left(x + \frac{n\pi}{2}\right).$$

which is obviously an identity.

Hence (B) is an identity, except for an additive constant. This constant must be zero, as we see if we let x approach zero. (A) follows from (B). Then since the given formula holds for $n = 0$, it holds for all values of n.

Formula (2) may be derived in the same manner.

Also solved by A. M. Harding, W. C. Eells and the Proposer.

NEWS AND NOTES.

FLORIAN CAJORI, CHAIRMAN OF THE COMMITTEE.

The March number of *Popular Science Monthly* contains an article on "Henri Poincaré as an Investigator" by PROFESSOR JAMES B. SHAW.

PROFESSOR G. A. MILLER, of the University of Illinois, will teach in the summer session of the University of California.